UKMT Maths Problems

None of these statements is true

Exactly one of these statements is true.

Exactly two of these statements are true.

All of these statements are true.

How many of the statements in the box are true?

Junior Mathematical Challenge, 2005.

United Kingdom Mathematics Trust

Number 1

UKMT Maths Solution

1

Each statement contradicts every other statement. So at most one of them can be true. If no statement is true, the first statement would be true. This would be a contradiction. Therefore exactly one statement - the second statement - is true.

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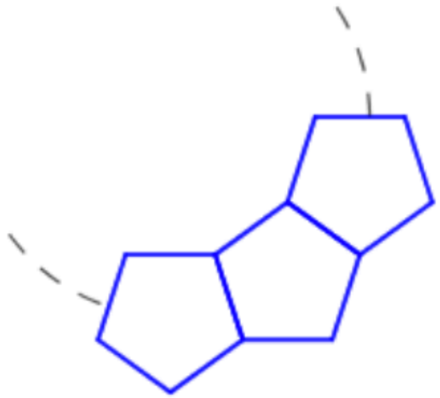
Number One

UKMT Maths Problem



Congruent regular pentagons are placed together to form a ring in the manner shown. The diagram shows the first three pentagons.

How many *more* pentagons are needed to complete the ring?

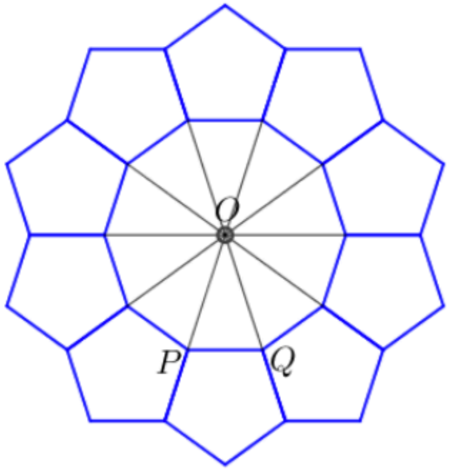
Intermediate Mathematical Challenge, 2001.

United Kingdom Mathematics Trust

Number Two

UKMT Maths Solution

7



The exterior angles of a regular pentagon are each $(360 \div 5)^\circ = 72^\circ$. Therefore $\angle OPQ = \angle OQP = 72^\circ$. Because the sum of the angles in a triangle is 180°, it follows that $\angle POQ = 180^\circ - 72^\circ - 72^\circ = 36^\circ$, and likewise for the other angles at O. Because the sum of angles at O is 360°, there are 10 of these angles. Hence the ring contains 10 pentagons. So the number of pentagons needed to complete the ring is 7.

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Number Two

UKMT Maths Problem

Earlier this year, the White Rabbit said to me, "Two days ago, Alice was still thirteen, but her sixteenth birthday will be next year."

When is Alice's birthday?

Senior Mathematical Challenge 1999,
images by Sir John Tenniel, from
Alice's Adventures in Wonderland.

United Kingdom Mathematics Trust

Number Three

UKMT Maths Solution

31st December

The White Rabbit was speaking on January 1st, this year. Two days earlier, 30th December, Alice was still thirteen. Her fourteenth birthday was the following day. She will be fifteen on 31st December this year, and sixteen on 31st December next year.

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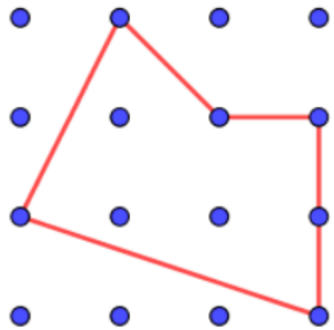
Number Three

UKMT Maths Problems



A single polygon is made by joining dots in the 4×4 grid with straight lines which meet only at dots at their end points. No dot is at more than one corner.

The diagram shows a five-sided polygon formed in this way.

What is the greatest possible number of sides of of a polygon drawn in the grid using these rules?

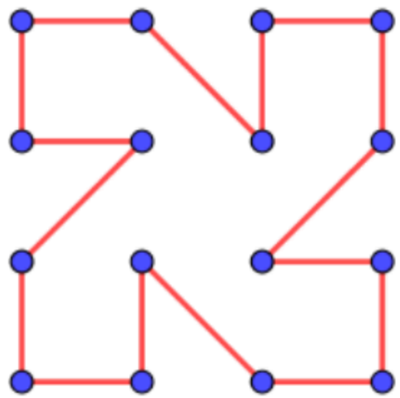
Junior Mathematical Challenge, 2010.

United Kingdom Mathematics Trust

Number Four

UKMT Maths Solution

16



There are 16 dots in the grid. No dot is at more than one corner. So no polygon drawn according to the rules has more than 16 corners, and therefore no more than 16 sides.

The diagram shows one way to draw a polygon according the rules with 16 sides. Therefore 16 is the greatest possible number of sides in such a polygon.

UKMT Maths Problem

$$\begin{array}{r} f\;l\;y \\ +\;f\;l\;y \\ +\;f\;l\;y \\ \hline a\;w\;a\;y \\ \hline \end{array}$$

In this addition sum, each letter represents a different non-zero digit?
Which number is represented by $a\ w\ a\ y$?

Intermediate Mathematical Challenge, 2004.

United Kingdom Mathematics Trust

Number Five

UKMT Maths Solution

2625

From the units column, as $y \neq 0$, there must be a carry of either 1 or 2 to the tens column. Hence $3y = 10 + y$ or $3y = 20 + y$. Therefore either $y = 5$ or $y = 10$. Since $y < 10$, we deduce that $y = 5$. Since $f\ l\ y \leq 999$, we have $a\ w\ a\ y < 3000$. Hence a is either 1 or 2. So, from the tens column, $3l + 1$ is either 1, 2, 11, 12, 21 or 22. Of these only 22 can be written as $3l + 1$ where l is a positive integer. It follows that $l = 7$, and there is a carry of 2 to the hundreds column. Now, from the hundreds column, we see that $3f + 2 = 20 + w$. Hence, either $f = 7, w = 3$ or $f = 8, w = 6$ or $f = 9, w = 9$. Since $f \neq l$ and $f \neq w$ we have $f = 8, w = 6$. We conclude that $a\ w\ a\ y = 2625$ (with $f\ l\ y = 875$).

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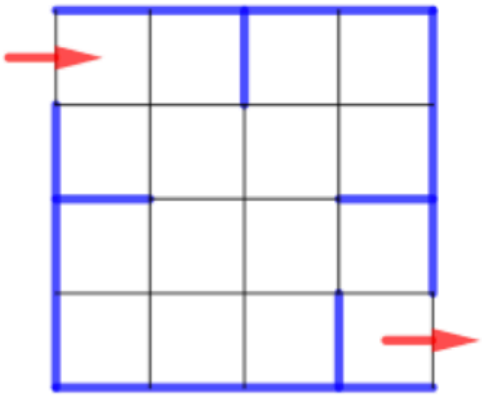
Number Five

UKMT Maths Problem



The diagram represents a maze.

The only moves allowed are to go from one cell to the next across an edge. You cannot cross the thick lines, and you cannot revisit a cell.

How many different routes are there through the maze?

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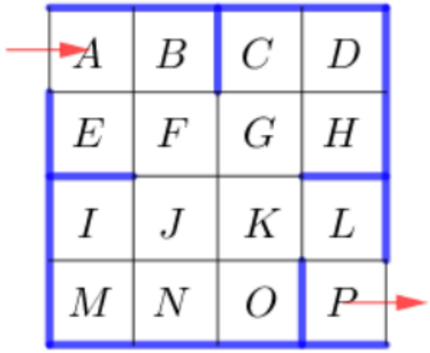
Number Six

UKMT Maths Solution

8



Each route from A from P must go through F and K.
There are two routes from A to F: $A \to B \to F$ and $A \to E \to F$, four routes from F to K: $F \to J \to K$, $F \to J \to I \to M \to N \to O \to K$, $F \to J \to N \to O \to K$ and $F \to G \to K$, and just one route from K to P: $K \to L \to P$.
Hence there are $2 \times 4 \times 1 = 8$ routes from A to P.

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Number Six

π Pi Week! π

Solution: The tip of the minute hand sweeps out the circumference of a circle of radius 7 cm each hour of the day, so it travels $24 \times 2\pi \times 7$, or 336π cm. The tip of the hour hand sweeps out the circumference of a circle of radius 6 cm twice in a day, so it travels $2 \times 2\pi \times 6$, or 24π cm. Total distance travelled by the tips of both hands is therefore **360π cm**.

UKMT

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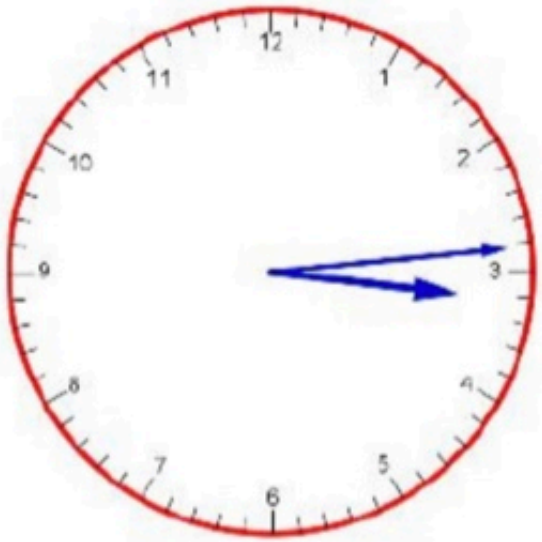
π Pi Week! π



An analogue clock has a minute hand that is 7cm in length, and an hour hand that is 6cm in length.

During a 24-hour day, how far in total do the tips of both hands travel?

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